

An analytical step is called an optimization if it occurs after the arrival time data $\{t_1, t_2, \dots, t_k\}$ are collected.

The purpose of mitigation steps is to reduce the error variances.

The purpose of optimization steps is to increase the likelihood of an accurate TPR.

An irregularly occurring, non-analytical step taken at any time to accomplish the same goals as mitigation and optimization is called ad-hoc.

The collection of ad-hoc, mitigation, or optimization steps taken in an implementation of the OSMMTS is called the system's containment policies, and referred to individually as a system containment policy.

The OSMMTS Demerit System is an ad-hoc containment policy that acts simultaneously as a mitigation and an optimization. Under this system, the three SDU's chosen to calculate the TPR are those three that are most likely to produce the "best" TPR based on past performance (thereby making it a optimization step), by way of reducing the variability of the utilized data (thereby making it a mitigation step).

Suppose there are n -many SDU's, however, only $k \leq n$ many receive a signal within the reception window. There are $\binom{n}{k}$ -many combinations of SDU's, and $\binom{k}{3}$ -many combinations of the k -many that receive the signal taken three at a time. Each SDU has three values associated with it at the beginning of each processing cycle, namely its non-negative Demerit Count, its positive History Total, and its possibly null Boolean Confirmation Value. At the beginning of all processing, the demerit count for each SDU will be zero, the history total will be one, and the confirmation value will be null. The confirmation value at the beginning of the processing cycle is determined by its observed value during the confirmation cycle. At the end of a processing cycle, the demerit count and history total are determined by the steps described in Claim 5(b), and the confirmation value is set back to null.

See also the *The Optimal Surface Mitigated Multiple Targeting System Patent Application Technical Documentation* memorandum referred to in the Information Disclosure Statement for a complete analytical description and derivation of the OSMMTS methods and processes.

5 Claims

1. A method for

- (a) Analytically calculating a Target Position Report (x_0, y_0) for arbitrarily many self-identifying targets in a two-dimensional grid according to the algorithm:

Given the speed of signal propagation c , Surface Detection Unit positions (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , and observed arrival time data $\{t_1, t_2, t_3\}$ at Surface Detection Unit positions (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , then

- i. Calculate $f_1 = t_1 - t_2$ and $f_2 = t_1 - t_3$.

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ii. Calculate $d_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ and $d_2 = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2}$.

iii. When $y_1 \neq y_2$, $y_1 \neq y_3$, and $d_i > |f_i|c$, for $i = 1, 2$, then

A. Calculate $m_1 = -\left(\frac{x_2 - x_1}{y_2 - y_1}\right)$.

B. Calculate $m_2 = -\left(\frac{x_3 - x_1}{y_3 - y_1}\right)$.

C. Use $b_1 = \frac{f_1 c \sqrt{4\kappa + (f_1 c)^2 + (x_2^2 - x_1^2) + (y_2^2 - y_1^2)}}{2(y_2 - y_1)}$.

D. Use $b_2 = \frac{f_2 c ((f_1 + f_2)c + \sqrt{4\kappa + (f_1 c)^2 + (x_2^2 - x_1^2) + (y_2^2 - y_1^2)})}{2(y_3 - y_1)}$.

E. Solve

$$\kappa = \frac{d_1^2 - (f_1 c)^2}{2} + \left\{ \begin{aligned} &\left(x_1 - \left(\frac{b_2 - b_1}{m_1 - m_2}\right)\right) \left(x_2 - \left(\frac{b_2 - b_1}{m_1 - m_2}\right)\right) \\ &+ \left(y_1 - \left(m_1 \left(\frac{b_2 - b_1}{m_1 - m_2}\right) + b_1\right)\right) \left(y_2 - \left(m_1 \left(\frac{b_2 - b_1}{m_1 - m_2}\right) + b_1\right)\right) \end{aligned} \right\}$$

for κ ; call this value κ_0 .

F. Evaluate b_1 and b_2 with $\kappa = \kappa_0$; call these values β_1 and β_2 , respectively.

G. Then

$$(x_0, y_0) = \left(\frac{\beta_2 - \beta_1}{m_1 - m_2}, m_1 \left(\frac{\beta_2 - \beta_1}{m_1 - m_2} \right) + \beta_1 \right)$$

iv. When $y_1 = y_2$, $y_1 \neq y_3$, and $d_i > |f_i|c$, for $i = 1, 2$, then

A. Use $r_1 = \frac{\pm f_1 c \sqrt{4\kappa + (f_1 c)^2 + (x_2^2 - x_1^2)}}{2(x_2 - x_1)}$.

B. Use $r_2 = \frac{1}{2(y_3 - y_1)} \left(\begin{aligned} &f_2 c \left((f_1 + f_2)c \pm \sqrt{4\kappa + (f_1 c)^2} \right) + (x_3^2 - x_1^2) + (y_3^2 - y_1^2) \\ &- \left(\pm f_1 c \sqrt{4\kappa + (f_1 c)^2 + (x_2^2 - x_1^2)} \right) \left(\frac{x_3 - x_1}{x_2 - x_1} \right) \end{aligned} \right)$.

C. Solve

$$\kappa = \frac{d_1^2 - (f_1 c)^2}{2} + (x_1 - r_1)(x_2 - r_1) + (y_1 - r_2)(y_2 - r_2)$$

for κ ; call this value κ_0 .

D. Evaluate r_1 and r_2 with $\kappa = \kappa_0$; call these values γ_1 and γ_2 , respectively.

E. Then $(x_0, y_0) = (\gamma_1, \gamma_2)$.

v. When $y_1 \neq y_2$, $y_1 = y_3$, and $d_i > |f_i|c$, for $i = 1, 2$, then

A. Use $r_1 = \frac{f_2 c^2 (f_1 + f_2) \pm \sqrt{4\kappa_0 + (f_1 c)^2 + (x_3^2 - x_1^2)}}{2(x_3 - x_1)}$.

B. Use $r_2 = \frac{1}{2(y_2 - y_1)} \left(\begin{aligned} &\pm f_1 c \sqrt{4\kappa_0 + (f_1 c)^2 + (x_2^2 - x_1^2) + (y_2^2 - y_1^2)} \\ &- \left(f_2 c \left((f_1 + f_2)c \pm \sqrt{4\kappa_0 + (f_1 c)^2} \right) + (x_3^2 - x_1^2) \right) \left(\frac{x_2 - x_1}{x_3 - x_1} \right) \end{aligned} \right)$.

C. Solve

$$\kappa = \frac{d_1^2 - (f_1 c)^2}{2} + (x_1 - r_1)(x_2 - r_1) + (y_1 - r_2)(y_2 - r_2)$$

for κ ; call this value κ_0 .

D. Evaluate r_1 and r_2 with $\kappa = \kappa_0$; call these values γ_1 and γ_2 , respectively.

E. Then

$$(x_0, y_0) = (\gamma_1, \gamma_2)$$

2. A system, comprising

- (a) one Principal Application Specific Integrated Circuit Central Processing Unit that implements said method of Claim 1.
- (b) a set of (at least three) Surface Detection Units that send information to said Principal Application Specific Integrated Circuit Central Processing Unit.
- (c) a network of databases of statically stored data that said Principal Application Specific Integrated Circuit Central Processing Unit uses to produce said Target Position Report.

3. A system of Claim 2, wherein said information sent from said Surface Detection Units to said Principal Application Specific Integrated Circuit Central Processing Unit is uniquely coded in a format that

- (a) said Principal Application Specific Integrated Circuit Central Processing Unit uses to identify said communicating Surface Detection Unit.
- (b) said network of databases of statically stored data uses to update its data.

4. A method of Claim 1, wherein said step of calculating said Target Position Report provides

- (a) an Error Likelihood Ellipse, given by those points (x_0^*, y_0^*) , such that

$$\left(\frac{x_0^* - x_0}{1 - \frac{m_1}{m_2}} \right)^2 + \left(\frac{y_0^* - y_0}{m_1 - m_2} \right)^2 = R$$

where $(m_1 - m_2)(x_0^* - x_0)$ is distributed as $M_2(t_1, t_2, t_3)$, and $\left(1 - \frac{m_2}{m_1}\right)(y_0^* - y_0)$ is distributed as $M_2(t_1, t_2, t_3)$, and $m_1 = -\left(\frac{x_2 - x_1}{y_2 - y_1}\right)$ and $m_2 = -\left(\frac{x_3 - x_1}{y_3 - y_1}\right)$, and $M_2(t_1, t_2, t_3)$ is the probability distribution given by the difference between two independent $M_1(t_1, t_2, t_3)$ distributions, and for which, in turn, $M_1(t_1, t_2, t_3)$ is a probability distribution

given by $\frac{1}{2(v_3 - v_1)} \left(\mp f_2 c \sqrt{2 \left(\frac{d_2^2 - (f_2 c)^2}{(1 - \cos \theta_0^*)} \right)} + (f_2 c)^2 \pm \xi_2 \sqrt{2 \left(\frac{d_2^2 - \xi_2^2}{(1 - \cos \theta_0^*)} \right)} + \xi_2^2 \right)$

(where the choice of sign depends on the relationships between (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , as in Claim 1), and where $\xi_2 = (f_2 - (\epsilon_1 - \epsilon_3))c$ is distributed as $N(f_2 c, 2c^2 \sigma^2)$ (the standard normal distribution with σ being the standard deviation of the errors occurring in the system described in Claim 3(b)), and η is distributed as $M_0(t_1, t_2, t_3)$, where $M_0(t_1, t_2, t_3)$ is a probability distribution given by $\cos \theta_0^* - \cos \theta_0$, where θ_0 is the (acute) angle of intersection between the lines of intersection L_1 and L_2 , where

$$L_1 : \begin{cases} x = \frac{\pm f_1 c \sqrt{4\kappa_1 + (f_1 c)^2} + (x_2^2 - x_1^2)}{2(x_2 - x_1)}, & y_1 = y_2 \\ y = -x \left(\frac{x_2 - x_1}{y_2 - y_1} \right) + \left[\frac{\pm f_1 c \sqrt{4\kappa_1 + (f_1 c)^2} + (x_2^2 - x_1^2) + (y_2^2 - y_1^2)}{2(y_2 - y_1)} \right], & y_1 \neq y_2 \end{cases}$$

and

$$L_2 : \begin{cases} x = \frac{f_2 c ((f_1 + f_2) c \pm \sqrt{4\kappa_1 + (f_1 c)^2}) + (x_3^2 - x_1^2)}{2(x_3 - x_1)}, & y_1 = y_3 \\ y = -x \left(\frac{x_3 - x_1}{y_3 - y_1} \right) + \left[\frac{f_2 c ((f_1 + f_2) c \pm \sqrt{4\kappa_1 + (f_1 c)^2}) + (x_3^2 - x_1^2) + (y_3^2 - y_1^2)}{2(y_3 - y_1)} \right], & y_1 \neq y_3 \end{cases}$$

(with corresponding values for θ_0^*), where κ_1 is the value of κ_0 found in Claim 1(a)(iii)(E), or Claim 1(a)(iv)(C), or Claim 1(a)(v)(C), when combinations of $\{(x_1, y_1), (x_2, y_2)\}$ (for L_1) and $\{(x_1, y_1), (x_3, y_3)\}$ (for L_2) are used, and the Standardized Elliptical Constant R is defined by

$$R = \frac{(m_1 - m_2)(x_0^* - x_0)^2 + \left(1 - \frac{m_2}{m_1}\right)(y_0^* - y_0)^2}{\left(\frac{(m_1 - m_2)^2}{m_1}\right)}$$

- (b) a Likelihood of Accuracy measure of said Target Position Report using said Error Likelihood Ellipse, calculated by the area of

$$(m_1 - m_2)(x_0^* - x_0)^2 + \left(1 - \frac{m_2}{m_1}\right)(y_0^* - y_0)^2 = \frac{(m_1 - m_2)^2}{m_1} R$$

with all terms as defined in Claim 4(a).

5. A method of Claim 1, wherein said step of calculating said Target Position Report uses

- (a) arrival times at a said set of Surface Detection Units.
- (b) a Demerit System, implemented as follows: For each processing cycle, and for each of the $\binom{k}{3}$ -many combinations, the following steps determine the end-of-processing-cycle demerit counts and history totals.

- i. Set the likelihood value λ .
- ii. Eliminate those τ_0 -many combinations of signal receiving Surface Detection Units that are collinear.
- iii. Eliminate those τ_1 -many combinations of signal receiving Surface Detection Units that do not all
 - A. have positive history totals, and
 - B. have received a confirmation during the Confirm phase of the current processing cycle.
- iv. The Surface Detection Units involved in the $(\tau_0 + \tau_1)$ -many combinations eliminated under the previous two steps are called deficient for the current processing cycle. This designation is removed at the beginning of a new processing cycle.
- v. Among the remaining, that is, qualifying combinations, choose the combination of three Surface Detection Units that collectively have the minimal sum of demerits.
- vi. In case of a tie in the previous step, use the combination with the largest history sum. In case of a further tie, choose the combination with the smallest individual demerit count. In case of a last tie, randomly choose uniformly among the finalists. The combination so chosen is called the calculating combination, and the Surface Detection Units involved are called the elected Surface Detection Units. Increment the history total by 1 for each elected Surface Detection Unit.
- vii. Subtract two demerits from the count for each elected Surface Detection Unit. Recall the demerit count for an Surface Detection Unit cannot become negative.
- viii. Calculate the Target Position Report using the calculating combination.
- ix. Calculate the λ -Error Likelihood Ellipse for the calculating combination, which is the Error Likelihood Ellipse described in Claim 4(a) that has a Likelihood of Accuracy value (found in Claim 4(b)) of λ .
- x. Calculate the $\binom{k}{3} - (\tau_0 + \tau_1)$ -many Target Position Reports for all other qualifying combinations. Each of these Target Position Reports is called an Alternate Position Report.
- xi. For each Alternate Position Report calculated in the previous step, if the Alternate Position Report falls outside the λ -Error Likelihood Ellipse, then add one demerit to the count for each Surface Detection Unit involved in the Alternate Position Report.
- xii. For each Alternate Position Report referred to in the previous step, if the Alternate Position Report falls inside or on the λ -Error Likelihood Ellipse, then subtract one demerit from the count for each Surface Detection Unit involved in the Alternate

Position Report. Recall the demerit count for a Surface Detection Unit cannot become negative.

- xiii. Add one demerit for each Surface Detection Unit that does not report a positive confirmation.
- xiv. When the demerit count for an Surface Detection Unit exceeds the Warning Threshold, send an alert to report a frequently deficient Surface Detection Unit.
- xv. When the demerit count for an Surface Detection Unit exceeds the Terminal Threshold, shut down communication with the Surface Detection Unit and do not consider it further (by setting its history total to zero) until explicitly reset. Also send an alert to report a failed Surface Detection Unit.
- xvi. These steps are in addition to the disabling of an Surface Detection Unit if proper query responses are not confirmed during the receive phase.

(c) a containment policy to maximize said Likelihood of Accuracy, which consists of calculating

$$\frac{d_{\min}}{c} = \frac{1}{c} \min_{0 \leq s \leq 1} \{d_1(s) + d_2(s)\} \text{ and } \frac{d_{\max}}{c} = \frac{1}{c} \min_{0 \leq s \leq 1} \{d_1(s) + d_2(s)\}$$

subject to

$$d^2(s) = d_1^2(s) + d_2^2(s) - 2d_1(s)d_2(s)\gamma(s)$$

where

$$d_1(s) = \sqrt{((x_1 - x_l) - s(x_r - x_l))^2 + ((y_1 - y_l) - s(y_r - y_l))^2}$$

and

$$d_2(s) = \sqrt{((x_0 - x_l) - s(x_r - x_l))^2 + ((y_0 - y_l) - s(y_r - y_l))^2}$$

and comparing $\frac{d_{\min}}{c}$ and $\frac{d_{\max}}{c}$ to any signal arrival time t_i at a particular Surface Detection Unit, and if

$$t_i < \frac{d_{\min}}{c} \text{ or } \frac{d_{\max}}{c} < t_i$$

then all Target Position Reports associated with the particular Surface Detection Unit are rejected, and a new Target Position Report is calculated using the next highest priority qualifying combination of Surface Detection Units. This cycle continues until a Target Position Report is accepted or the qualifying combinations of Surface Detection Units are depleted.

6. A system of Claim 2, wherein said Surface Detection Units are

- (a) physically distinct from said Principal Application Specific Integrated Circuit Central Processing Unit.